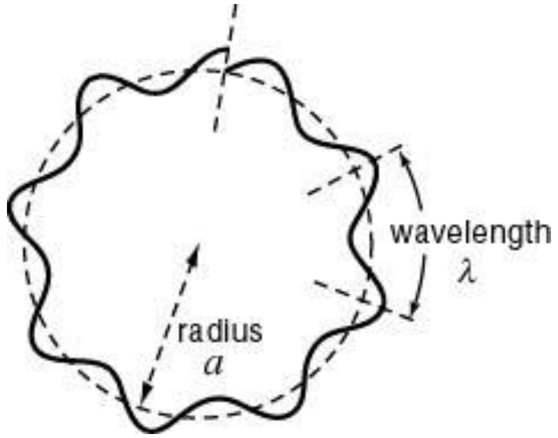
Wave Mechanical Atom Model

The wavelength associated with a particle, say an electron, is referred to as de Broglie wavelength. The electrons move around the nucleus as wave packets, which are formed in a somewhat similar manner as standing waves are formed in sound.

Consider the electron in the hydrogen atom as a standing wave extending in a circle round the nucleus. In order that this wave may just occupy the circumference of a circle, the circle must contain an integral number of wavelengths. If r is th radius of the circular orbit, then

$$2\pi r = n\lambda = n\frac{h}{mv} \Rightarrow mvr = n\frac{h}{2\pi}$$



But, *mvr* is the angular momentum of the electron as a particle. Thus, the angular momentum is equal to an integral multiple of *h*/2π.

The Heisenberg Uncertainty Principle

- Whenever a measurement is made there is always some uncertainty
- Quantum mechanics limits the accuracy of certain measurements because of wave – particle duality and the resulting interaction between the target and the detecting instrument

The Heisenberg Uncertainty Principle

The **Heisenberg uncertainty principle** states that it is impossible to know both the momentum and the position of a particle at the same time.

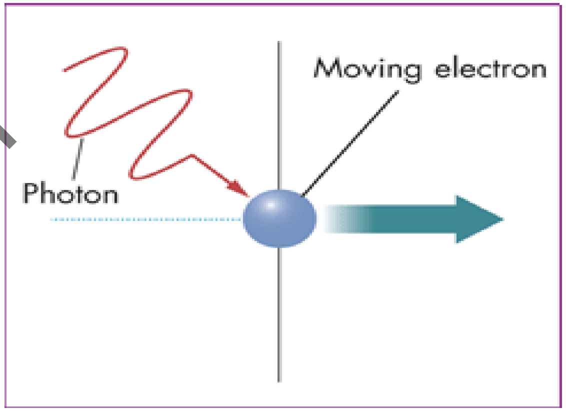
- This limitation is critical when dealing with small particles such as electrons.
- But it does not matter for ordinary-sized objects such as cars or airplanes.

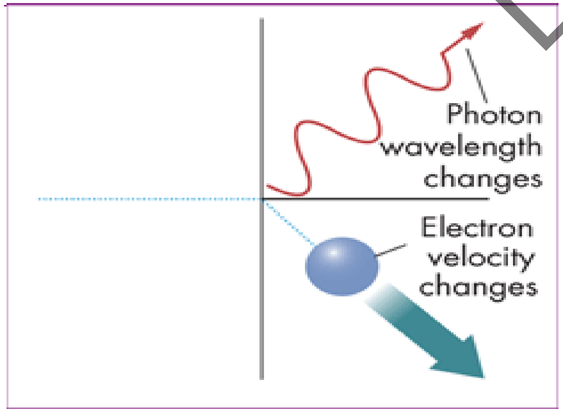
The Heisenberg Uncertainty Principle

- To locate an electron, you might strike it with a photon.
- The electron has such a small mass that striking it with a photon affects its motion in a way that cannot be predicted accurately.
- The very act of measuring the position of the electron changes its momentum, making its momentum uncertain.

The Heisenberg Uncertainty Principle

Before collision: A photon strikes an electron during an attempt to observe the electron's position.





- **After collision:** The impact changes the electron's momentum, making it uncertain.

The Heisenberg Uncertainty Principle

- If we want accuracy in position, we must use short wavelength photons because the best resolution we can get is about the wavelength of the radiation used. Short wavelength radiation implies high frequency, high energy photons. When these collide with the electrons, they transfer more momentum to the target. If we use longer wavelength, i.e less energetic photons, we compromise resolution and position.

The Heisenberg Uncertainty Principle

- Symbolically $\Delta x \approx \lambda$
- $\Delta p \approx h/\lambda$
- $\Delta p\lambda \approx h$
- $(\Delta p)(\Delta x) \approx h$
- $(\Delta p)(\Delta x) \geq h/2\pi \geq \hbar$

The Heisenberg Uncertainty Principle

- The uncertainty principle can also relate energy and time as follows
- $\Delta t \approx \lambda/c$
- $\Delta E \approx hc/\lambda$
- $(\Delta E)(\Delta t) \approx h$
- $(\Delta E)(\Delta t) \geq h/2\pi \geq \hbar$

Problem: An electron moves in the x direction with a speed of 3.6×106 m/s. We can measure its speed to a precision of 1%. With what precision can we simultaneously measure its position?

The momentum of the electron along the x direction is,

$$p_x = mv_x = (9.1\times10^{-31}\,\mathrm{kg})(3.6\times10^{6}\,\mathrm{m/s}) = 3.3\times10^{-24}\,\mathrm{kg.m/s}$$

The uncertainty Δp_x is 1% of this value and is given by

$$= (3.3\times10^{-24})(0.01) = 3.3\times10^{-26}\,\mathrm{kg.m/s}$$

The uncertainty in its position is then given by the relation

$$\Delta x = \frac{\hbar}{\Delta p_x} = \frac{1.05\times10^{-34}\,J.s}{3.3\times10^{-26}\,kg.m/s} = 3.2\times10^{-9} = 3.2\,\mathrm{nm}$$

This is roughly equal to 10 atomic diameters.

Consider a pitched baseball (m=0.145 kg) to be moving with a speed of 42.5 m/s. Again it is assumed that the speed can be measured to a precession of 1%. With what precision can we simultaneously measure its position?

The momentum of the baseball is,

$$p_x = mv_x = (0.145\,\text{kg})(42.5\,\text{m/s}) = 6.16\,\text{kg.m/s}$$

The uncertainty Δp_x is 1% of this value and is given by

$$= (6.16)(0.01) = 0.0616\,\text{kg.m/s}$$

The uncertainty in its position is then given by the relation

$$\Delta x = \frac{\hbar}{\Delta p_x} = \frac{1.05\times10^{-34}\ J.s}{0.0616\,kg.m/s} = 1.7\times10^{-33}\ \text{m}$$

This is 19 orders of magnitude smaller than the size of an atomic nucleus.