Superposed vibrations in One Dimension

Many physical situations involve the simultaneous application of two or more harmonic vibrations to the same system. Example of this are specially common in acoustics. A phonograph stylus, a microphone diaphragms or a human eardrum is general being subjected to a complicated combination of such vibrations, resulting in some over all patterns of its displacements as a function of time.

Composition of two simple harmonic vibration in a straight line:

Let the two SHM be presented by the equations:

$$y_1 = a_1 \sin(\omega t + \alpha_1) -------(1)$$

$$y_2 = a_2 \sin(\omega t + \alpha_2) -------(2)$$

Where y_1 and y_2 are the displacements of a particle due to the two vibrations, a_1 and a_2 are the amplitudes and α_1 and α_2 are the epoch angles. Here, the two vibrations are assumed to be same frequency.

The resultant displacement y of the particle is given by

$$\mathrm{y} = \mathrm{y}_1 + \mathrm{y}_2$$

$$= (a_1 \cos\alpha_1 + a_2 \cos\alpha_2)\sin\omega t + (a_1 \sin\alpha_1 + a_2 \sin\alpha_2)\cos\omega t$$

Since the amplitudes a_1 and a_2 and the angles α_1 and α_2 are constants, the coefficient of sinωt and cosωt in equation (3) can be substituted by Acosϕ and Asinϕ.

$$\mathrm{A}\cos\phi = a_1 \cos\alpha_1 + a_2 \cos\alpha_2 ----(4)$$

$$\mathrm{A}\sin\phi = a_1 \sin\alpha_1 + a_2 \sin\alpha_2 ----(5)$$

$$\mathrm{A}^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\alpha_1 - \alpha_2) ----(6)$$

$$\tan\phi = \frac{a_1 \sin\alpha_1 + a_2 \sin\alpha_2}{a_1 \cos\alpha_1 + a_2 \cos\alpha_2} --------(7)$$

The resultant displacement y of the particle is given by

$$\mathrm{y} = A\cos\phi \sin\omega t + A\sin\phi \cos\omega t = A\sin(\omega t + \phi) ---(8)$$

Equation (8) is similar to the original equations (1) and (2). The amplitude of the resultant vibrations is A and epoch angle ϕ. Thus, the resultant of two SHM of the same period and acting in the same line is also a SHM with a resultant vibration A and epoch angle ϕ.

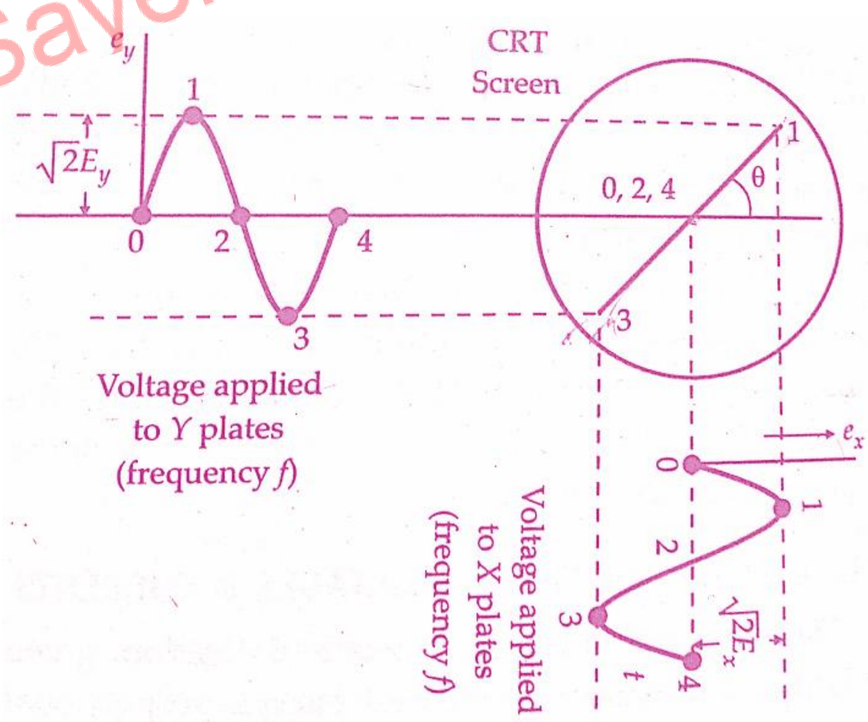
Special case: If $\alpha_1 = \alpha_2 = \alpha$, $A = a_1 + a_2$, $\phi = \alpha$

$$y = (a_1 + a_2)\sin(\omega t + \alpha)$$

Lissajous Figures:

When a particle is influenced simultaneously by two SHM at right angles to each other, the resultant motion of the particle traces a curve. These curves are called Lissajous figures. The shape of the curve depends on the time period, phase difference and the amplitude of the two constituent vibrations. Lissajous figures are helpful in determining the ratio of the time periods of two vibrations and to compare the frequencies of two tuning fork.

Lissajous figure formed by two sinusoidal SHM. SHM acted right angles to each other and their phase difference is zero degree.
Here, a circle is formed.



Some typical Lissajous Figures

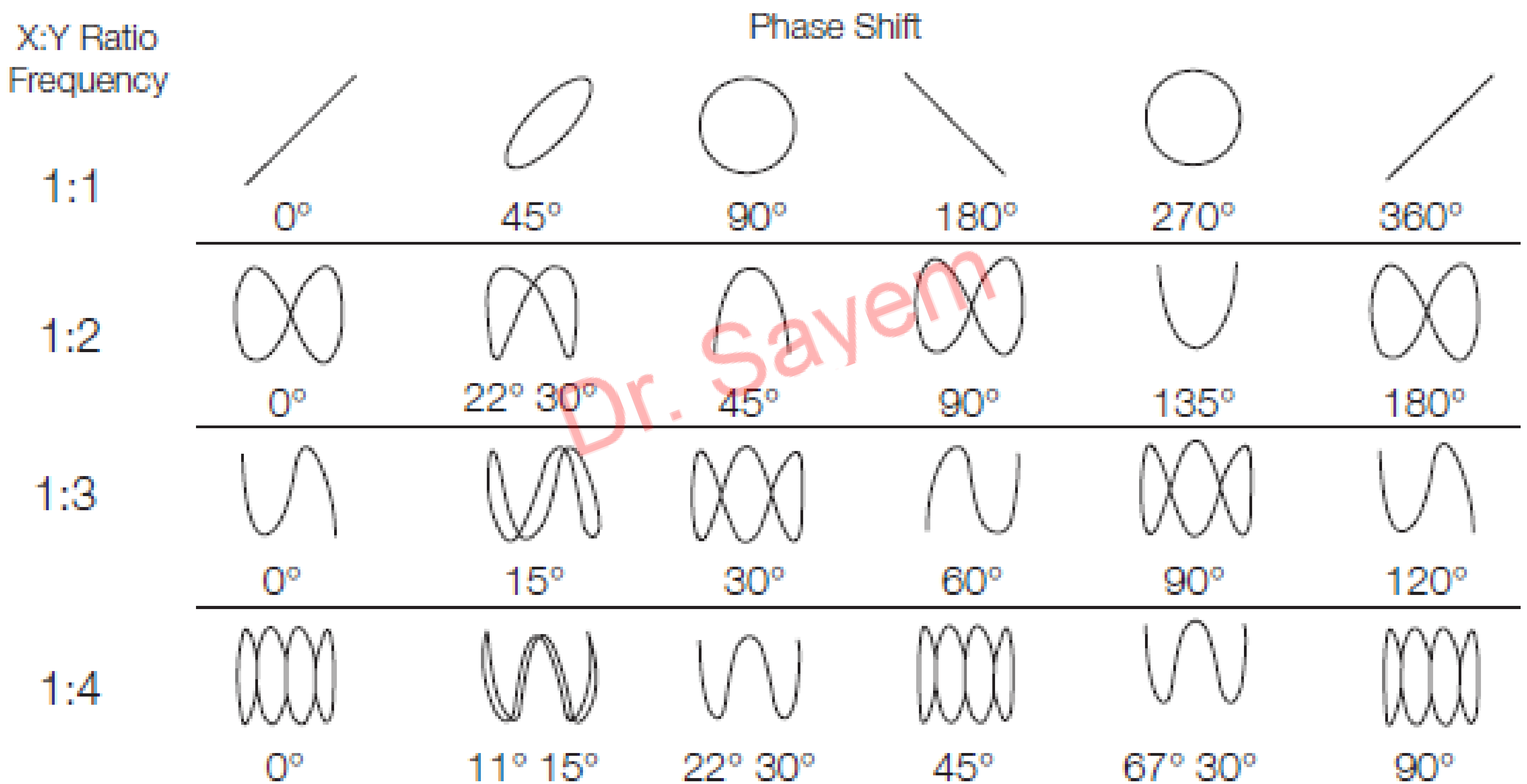
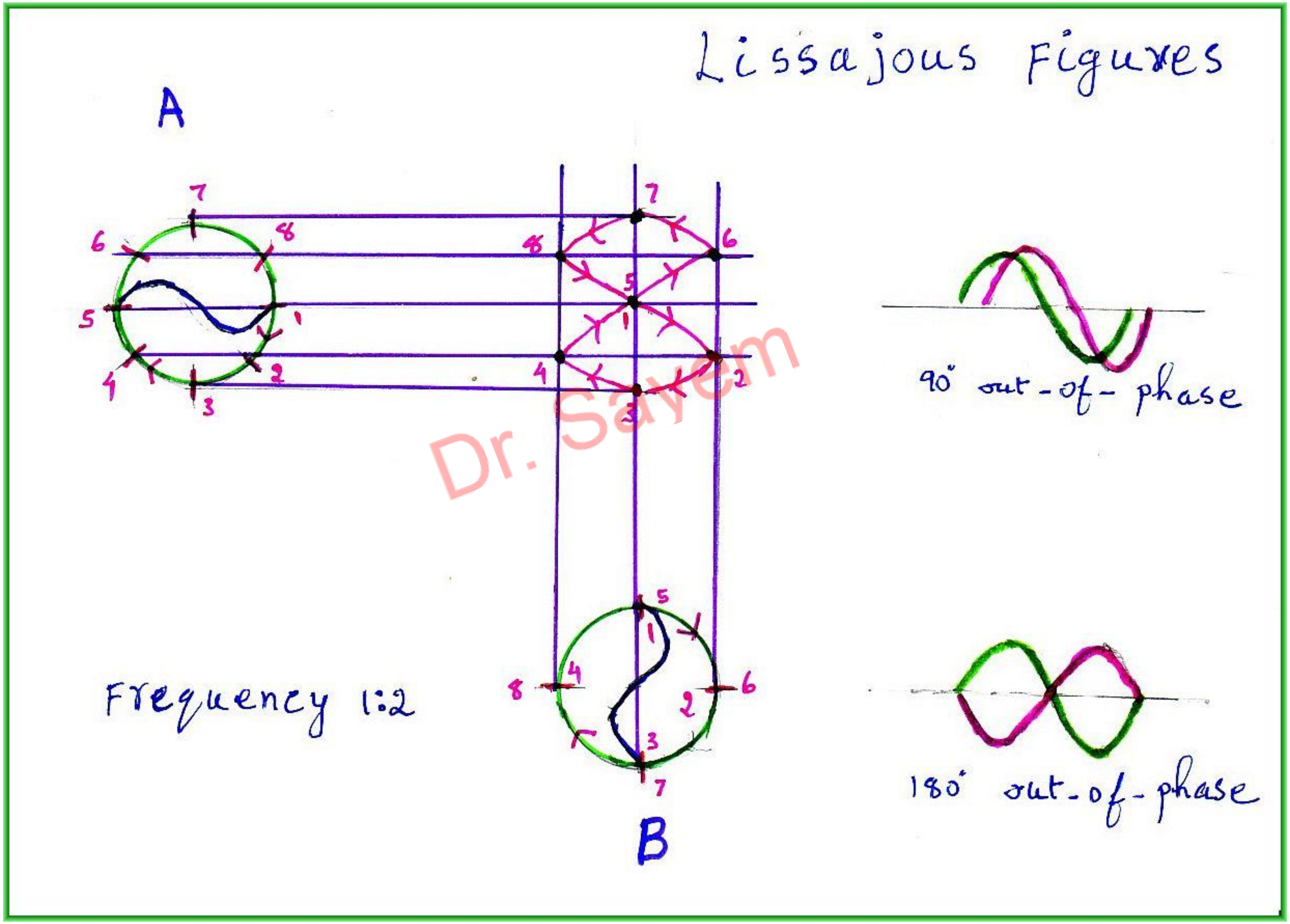
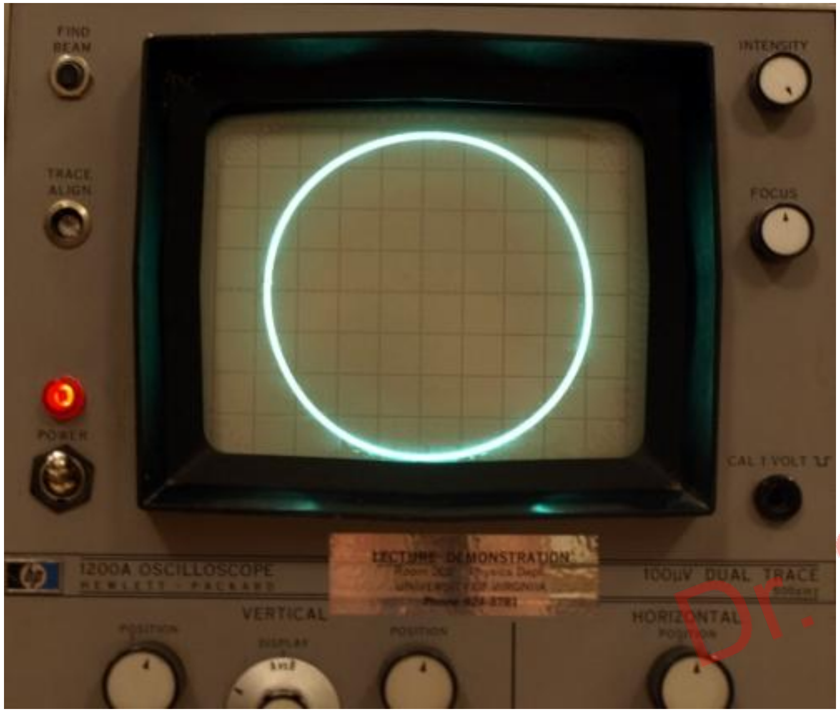


Figure 70. Lissajous patterns.

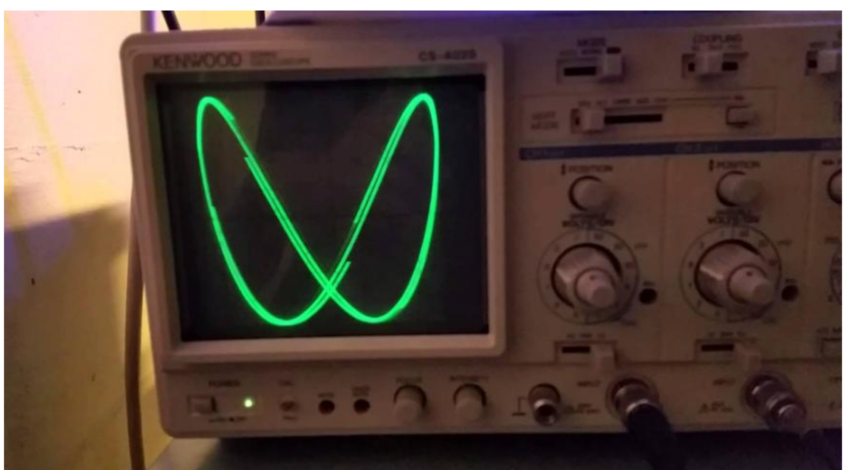
Lissajous Figure of frequency ratio 1:2

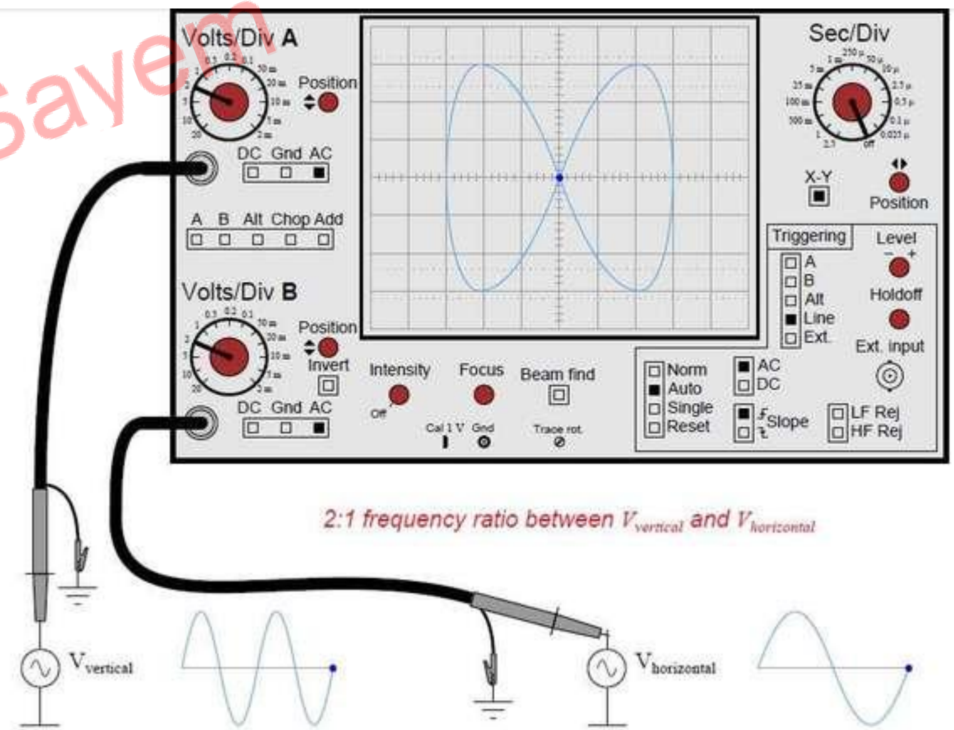


Lissajous Figure Produced in Oscilloscope



Two phase-shifted sinusoidal inputs are applied to the oscilloscope in X-Y mode and the phase relationship between the signals is presented as a Lissajous figure.





Uses of Lissajous figure

These figures allow one to compare amplitudes, frequencies and phase between two oscillatory signals for one.

For example, if you had two sinusoidal signals of equal amplitude and frequency, you could determine the phase difference by looking at the shape of the trace. If the waves are in phase, you would see a straight line, if the waves are pi/2 out of phase, you would see a circle instead.

In undergrad physics laboratory in BUET, it is used to find out the line frequency of the supply voltage with the help of signal generator that can produce several frequencies.