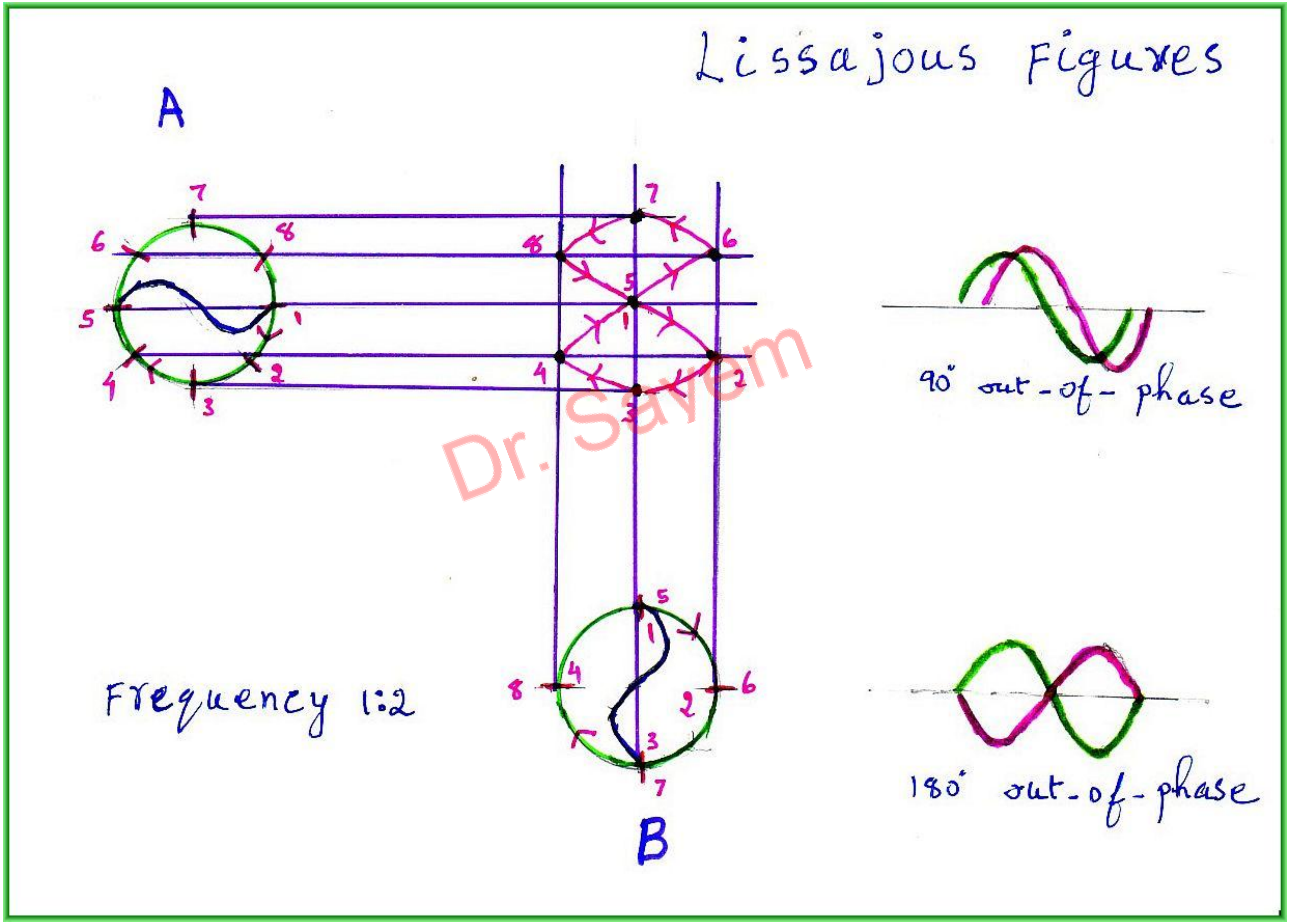
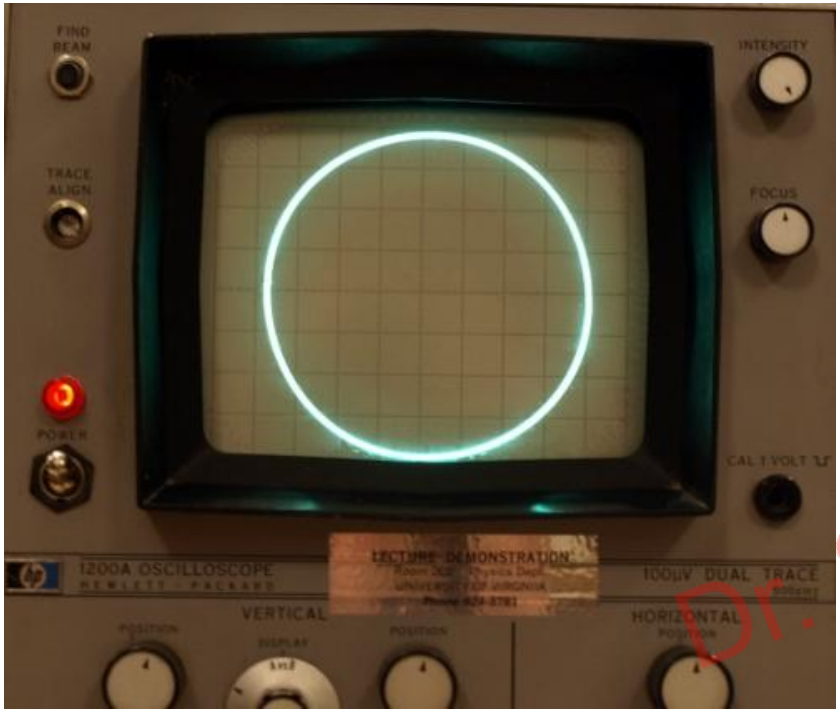
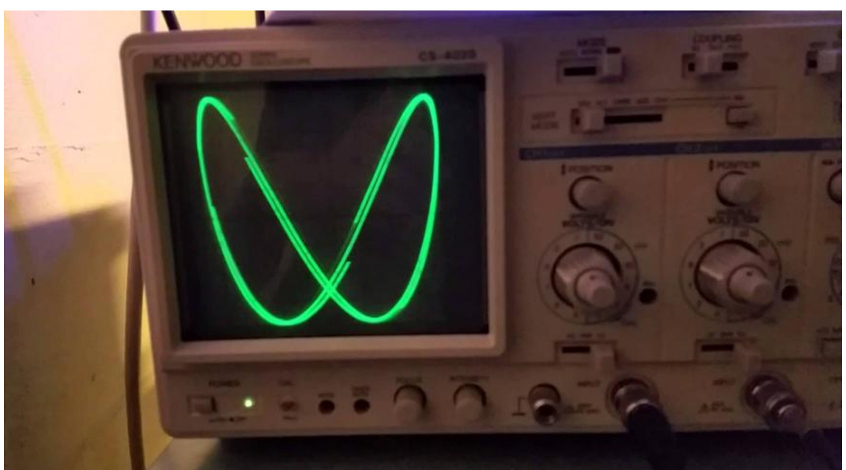
Lissajous Figure of frequency ratio 1:2

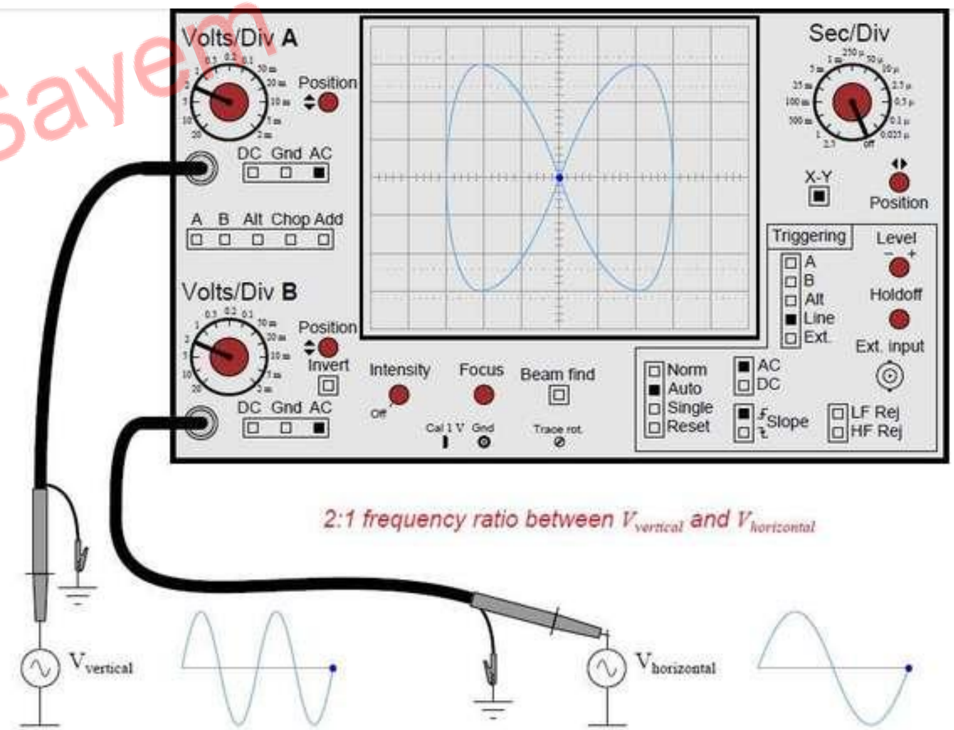


Lissajous Figure Produced in Oscilloscope





Two phase-shifted sinusoidal inputs are applied to the oscilloscope in X-Y mode and the phase relationship between the signals is presented as a Lissajous figure.



2:1 frequency ratio between $V_{vertical}$ and $V_{horizontal}$

Uses of Lissajous figure

These figures allow one to compare amplitudes, frequencies and phase between two oscillatory signals for one.

For example, if you had two sinusoidal signals of equal amplitude and frequency, you could determine the phase difference by looking at the shape of the trace. If the waves are in phase, you would see a straight line, if the waves are $\pi/2$ out of phase, you would see a circle instead.

In undergrad physics laboratory in BUET, it is used to find out the line frequency of the supply voltage with the help of signal generator that can produce several frequencies.

A particle performs SHM given by the equation, $y = 20\sin(\omega t + \alpha)$
If the time period is 30 s and the particle has a displacement of 10 cm at t =0, find (i) epoch; (ii) the phase angle at t= 5 s, and (iii) the phase difference between two positions of the particle 15 s apart.

Solution: $y = 20\sin(\omega t + \alpha)$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{30} = \frac{\pi}{15} rad$$

$(i) At, t = 0, y = 10\ cm$ $\quad 10 = 20\sin(\frac{\pi}{15} \times 0 + \alpha) \Rightarrow \alpha = \frac{\alpha}{6} rad$

$(ii) At, t = 5\ s,$ The phase angle $= (\omega t + \alpha) = (\frac{\pi}{15} \cdot 5 + \frac{\pi}{6}) = \frac{\pi}{2}$

$(iii) At, t = 0\ s,$ The phase angle $\theta_1 = \frac{\pi}{6}$

$At, t = 15\ s,$ The phase angle $\theta_2 = (\frac{\pi}{15} \cdot 15 + \frac{\pi}{6}) = \frac{7\pi}{6}$

The phase difference $= \theta_2 - \theta_1 = \frac{7\pi}{6} - \frac{\pi}{6} = \pi\ rad$

Two SHMs acting simultaneously on a particle are given by the equations:

$$y_1 = 2\sin(\omega t + \frac{\pi}{6})$$

$$y_2 = 3\sin(\omega t + \frac{\pi}{3})$$

Calculate (i) amplitude (ii) phase constant (iii) time period of the resultant vibration.

Solution: $A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\alpha_1 - \alpha_2)$

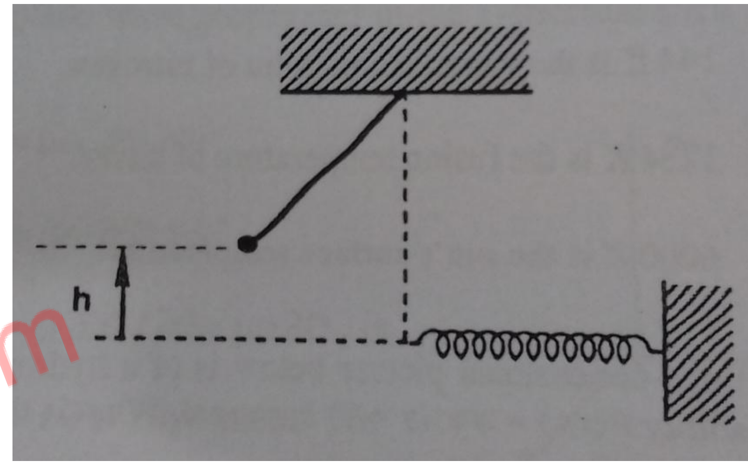
$$\Rightarrow A = 4.83$$

$$\tan\phi = \frac{a_1 \sin\alpha_1 + a_2 \sin\alpha_2}{a_1 \cos\alpha_1 + a_2 \cos\alpha_2} \Rightarrow \phi = 48.1^0 = 48.1 \times \frac{\pi}{180} = \frac{4\pi}{15} rad$$

$$\text{Phase constant} = (\omega\text{t} + \frac{4\pi}{14}) rad$$

(iii) The resultant time period is the same as the individual time period.

A pendulum of mass m raised to a height h and released. After hitting a spring of non-linear force law, $F=-kx-bx^3$ calculate the compression distance x of the spring.



Solution: By conservation of energy,

$$mgh=\frac{1}{2}mv^2 \Rightarrow v=\sqrt{2gh}$$

is the pendulum bob velocity just before it hits the spring.

The conservative force is, $F=-kx-bx^3$

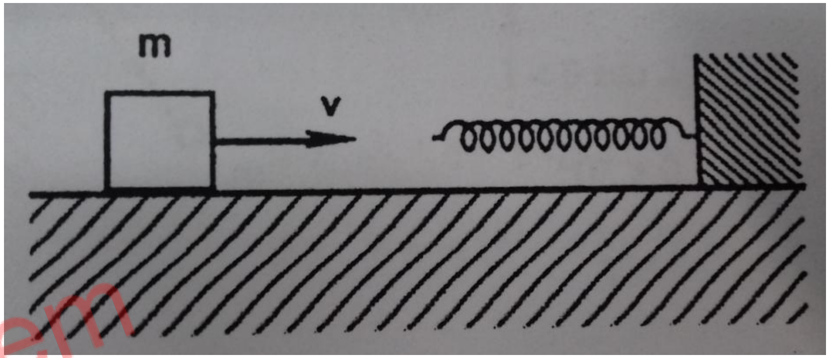
So that the potential energy, $U=-\int F dx=\frac{1}{2}kx^2+\frac{1}{4}bx^4$

Again by conservation of energy, $\frac{1}{2}mv^2=mgh=\frac{1}{2}kx^2+\frac{1}{4}bx^2$

Rearranging,

$$(x^2+\frac{k}{b})^2=\frac{4mgh}{b}+\frac{k^2}{b^2} \Rightarrow x=\left(\sqrt{\frac{4mgh}{b}+\frac{k^2}{b^2}}-\frac{k}{b}\right)^{1/2}$$

A block of mass m moving at speed v collides with a spring of restoring force $F=-k_1x-k_2x^3$ on a frictionless surface. Find the maximum compression of the spring.



Solution:

The given spring force is nonlinear but conservative, $F=-k_1x-k_2x^3$

Using the work-energy theorem:

$$W=\Delta K \Rightarrow \int F\cdot dx=\frac{1}{2}k_1x^2+\frac{1}{4}k_2x^4=\frac{1}{2}mv^2$$

$$\Rightarrow x^4+2\frac{k_1}{k_2}x^2=\frac{2m}{k_2}v^2 \Rightarrow (x^2+2\frac{k_1}{k_2})^2=\frac{2m}{k_2}v^2+\left(\frac{k_1}{k_2}\right)^2$$

$$x^2=-\frac{k_1}{k_2}+\sqrt{\frac{2m}{k_2}v^2+\left(\frac{k_1}{k_2}\right)^2} \Rightarrow x=\sqrt{\frac{k_1}{k_2}}\left(1+\frac{2mv^2k_2}{k_1^2}-1\right)^{1/2}$$